OTHER COLLEGE

FYJC - MATHEMATICS & STATISTICS

PAPER - I

CIRCLE

Compiled & Conducted @ JKSC

CONICS – CIRCLE

- ✓ Standard form of the circle : $x^2 + y^2 = r^2$

- ✓ Given centre C ≡ (h,k) & radius = r , equation of the circle can be generated using

  Center Radius form

  $(x - h)^2 + (y - k)^2 = r^2$

- ✓ Given $A(x_1,y_2)$, $B(x_2,y_2)$ are the ends of the diameter , equation of the circle can be generated using

  Diameter Form

  $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

- ✓ In general , equation of the circle

  $x^2 + y^2 + 2gx + 2fy + c = 0$, where

  $C \equiv (-g,-f)$ $R = \sqrt{g^2 + f^2 - c}$

Q1.

01. find the equation of the circle with center (2 , –3) and passing through (–1 , 2)

    ans : $x^2 + y^2 - 4x + 6y - 21 = 0$

02. find the equation of the circle with center (1 , –2) and passing through (5 , 3)

    ans : $x^2 + y^2 - 2x + 4y - 36 = 0$

03. find the equation of the circle with center (–2 , 3) and passing through (1 , 7)

    ans : $x^2 + y^2 + 4x - 6y - 12 = 0$

04. find the equation of the circle with center ($^1/_2$, $^3/_2$) and radius 3

    ans : $2x^2 + 2y^2 - 2x - 6y - 13 = 0$

Q2.

01. find equation of circle with radius 5 and concentric with circle $x^2 + y^2 + 4x - 6y = 0$

    ans : $x^2 + y^2 + 4x - 6y - 12 = 0$

02. find equation of circle with radius 5 and concentric with circle $x^2 + y^2 - 6x - 4y - 3 = 0$

    ans : $x^2 + y^2 - 6x - 4y - 12 = 0$

03. find equ. of circle concentric with $x^2 + y^2 - 2x - 6y - 7 = 0$ and area 616 sq. units

    ans : $x^2 + y^2 - 2x - 6y - 186 = 0$

04. Find centre and radius of the circle : $3x^2 + 3y^2 - 12x - 18y - 11 = 0$

ans : $C(2, 3)$, $r = \sqrt{50/3}$

05. Find centre and radius of the circle : $2x^2 + 2y^2 - 2x - 8y - 13 = 0$

ans : $C(^1/_2, 2)$, $r = \sqrt{43} / 2$

06. Find centre and radius of the circle : $3x^2 + 3y^2 - 6x + 4y - 42 = 0$

ans : $C(1, ^{-2}/_3)$, $r = \sqrt{22} / 3$

07. find the center and the radius of the circle : $(x - 3)(x - 5) + (y - 1)(y - 7) = 0$

ans : $C(4, 4)$, $r = \sqrt{10}$

Q3.

01. find equation of the circle having centre (7,–2) and touching the x – axis

ans : $x^2 + y^2 - 14x + 4y + 49 = 0$

02. find equation of the circle having centre (–5,2) and touching the y – axis

ans : $x^2 + y^2 + 10x - 4y + 4 = 0$

03. find equation of the circle having radius = 1 and touching the x – axis at (–4,0)

ans : $x^2 + y^2 + 8x \pm 2y + 16 = 0$

Q4.

01. Find circle touching both the axes and having radius 7

ans : $x^2 + y^2 \pm 14x \pm 14y + 49 = 0$

02. find equation of the circle touching both axes and passing through (1,2)

ans : $x^2 + y^2 - 2x - 2y + 1 = 0$, $x^2 + y^2 - 10x - 10y + 25 = 0$

03. find equation of the circle touching both axes and passing through (–9,8)

ans : $x^2 + y^2 + 10x - 10y + 25 = 0$, $x^2 + y^2 + 58x - 58y + 841 = 0$

Q5.

01. Find equation of circle passing through (4 , 6) ; (–3 , 5) & (5 , –1)

ans : $x^2 + y^2 - 2x - 4y - 20 = 0$

02. Find equation of circle passing through (4 , 1) ; (–3 , –6) & (–2 , 1) **(MAR 2013)**

ans : $x^2 + y^2 – 2x + 6y – 15 = 0$

03. Find equation of circle passing through (4 , 1) ; (6 , 5) & whose center lies on $4x + y = 16$

ans : $x^2 + y^2 – 6x – 8y + 15 = 0$

Q6.

Show that following pair of circles touch each other . Find the point of contact

01. $x^2 + y^2 – 4x + 10y + 20 = 0$; $x^2 + y^2 + 8x – 6y – 24 = 0$

ans : circle touch each other externally , point of contact (1/5 , –13/5)

02. $x^2 + y^2 + 4x – 12y + 4 = 0$; $x^2 + y^2 – 2x – 4y + 4 = 0$

ans : circle touch each other internally , point of contact (8/5 , 6/5)

Q1. SOLUTION SET

01. find the equation of the circle with center (2 , –3) and passing through (–1 , 2)

SOLUTION

STEP 1 :

$$r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(2 + 1)^2 + (-3 - 2)^2}$$

$$= \sqrt{9 + 25}$$

$$= \sqrt{34}$$

STEP 2 : $C(2,-3)$, $r = \sqrt{34}$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 2)^2 + (y + 3)^2 = (\sqrt{34})^2$$

$$x^2 - 4x + 4 + y^2 + 6y + 9 = 34$$

$$x^2 + y^2 - 4x + 6y + 13 - 34 = 0$$

$$x^2 + y^2 - 4x + 6y - 21 = 0$$ ..... equation of the circle

02. find the equation of the circle with center (1 , –2) and passing through (5 , 3)

SOLUTION

STEP 1 :

$$r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(1 - 5)^2 + (-2 - 3)^2}$$

$$= \sqrt{16 + 25}$$

$$= \sqrt{41}$$

STEP 2 : $C(1,-2)$, $r = \sqrt{41}$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 1)^2 + (y + 2)^2 = (\sqrt{41})^2$$

$$x^2 - 2x + 1 + y^2 + 4y + 4 = 41$$

$$x^2 + y^2 - 2x + 4y + 5 - 41 = 0$$

$$x^2 + y^2 - 2x + 4y - 36 = 0$$ ..... equation of the circle

03. find the equation of the circle with center (–2 , 3) and passing through (1 , 7)

SOLUTION

STEP 1 :

$$r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(-2 - 1)^2 + (3 - 7)^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25} = 5$$

STEP 2 : $C(-2,3)$, $r = 5$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x + 2)^2 + (y - 3)^2 = (5)^2$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 25$$

$$x^2 + y^2 + 4x - 6y + 13 - 25 = 0$$

$$x^2 + y^2 + 4x - 6y - 12 = 0$$ ..... equation of the circle

04. find the equation of the circle with center (1/2 , 3/2) and radius 3

SOLUTION

STEP 1 : $C(1/2 , 3/2)$, $r = 3$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = 3^2$$

$$\left(\frac{2x - 1}{2}\right)^2 + \left(\frac{2y - 3}{2}\right)^2 = 9$$

$$\frac{4x^2 - 4x + 1}{4} + \frac{4y^2 - 12y + 9}{4} = 9$$

$$4x^2 + 4y^2 - 4x - 12y + 10 = 36$$

$$4x^2 + 4y^2 - 4x - 12y - 26 = 0$$

$$2x^2 + 2y^2 - 2x - 6y - 13 = 0$$ ..... equation of the circle

Q2.

01. find equation of circle with radius 5 and concentric with circle $x^2 + y^2 + 4x - 6y = 0$

SOLUTION

STEP 1 : $x^2 + y^2 + 4x - 6y = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = 4 \; ; \; 2f = -6$

$g = 2 \; ; \; f = -3 \; ; \; c = 0$

$C \equiv (-g , -f)$

$\equiv (-2 , 3)$

STEP 2 : $C(-2,3) \; , \; r = 5$

$(x - h)^2 + (y - k)^2 = r^2$

$(x + 2)^2 + (y - 3)^2 = (5)^2$

$x^2 + 4x + 4 + y^2 - 6y + 9 = 25$

$x^2 + y^2 + 4x - 6y + 13 - 25 = 0$

$x^2 + y^2 + 4x - 6y - 12 = 0$ ..... equation of the circle

02. find equation of circle with radius 5 and concentric with circle $x^2 + y^2 - 6x - 4y - 3 = 0$

SOLUTION

STEP 1 : $x^2 + y^2 - 6x - 4y - 3 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -6 \; ; \; 2f = -4$

$g = -3 \; ; \; f = -2 \; ; \; c = 0$

$C \equiv (-g , -f)$

$\equiv (3,2)$

STEP 2 : $C(3,2) \; , \; r = 5$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 3)^2 + (y - 2)^2 = (5)^2$

$x^2 - 6x + 9 + y^2 - 4y + 4 = 25$

$x^2 + y^2 - 6x - 4y + 13 - 25 = 0$

$x^2 + y^2 - 6x - 4y - 12 = 0$ ..... equation of the circle

03. find equ. of circle concentric with $x^2 + y^2 - 2x - 6y - 7 = 0$ and area 616 sq. units

SOLUTION

STEP 1 : $x^2 + y^2 - 2x - 6y - 7 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -2$; $2f = -6$

$g = -1$; $f = -3$; $c = 0$

$C \equiv (-g, -f) \equiv (1,3)$

STEP 2 : area = 616

$\pi r^2 = 616$

$r^2 = \frac{616}{\pi}$

$r^2 = \frac{616 \times 7}{22} = 196$

$r = 14$

STEP 3 : $C(1,3)$, $r = 14$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 1)^2 + (y - 3)^2 = (14)^2$

$x^2 - 2x + 1 + y^2 - 6y + 9 = 196$

$x^2 + y^2 - 2x - 6y + 10 - 196 = 0$

$x^2 + y^2 - 2x - 6y - 186 = 0$ ..... equation of the circle

04. Find centre and radius of the circle : $3x^2 + 3y^2 - 12x - 18y - 11 = 0$

SOLUTION

$3x^2 + 3y^2 - 12x - 18y - 11 = 0$

$x^2 + y^2 - 4x - 6y - \frac{11}{3} = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -4$; $2f = -6$; $c = {}^{-11}/_{3}$

$g = -2$; $f = -3$; $c = {}^{-11}/_{3}$

$C \equiv (-g, -f)$ $\quad R = \sqrt{g^2 + f^2 - c}$

$\equiv (2, 3)$ $\quad = \sqrt{4 + 9 + \frac{11}{3}}$

$= \sqrt{\frac{12 + 27 + 11}{3}}$

$= \sqrt{50/3}$

05. Find centre and radius of the circle : $2x^2 + 2y^2 - 2x - 8y - 13 = 0$

SOLUTION

$$2x^2 + 2y^2 - 2x - 8y - 13 = 0$$

$$x^2 + y^2 - x - 4y - \frac{13}{2} = 0$$

On comparing with

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$2g = -1 \; ; \quad 2f = -4 \; ; \quad c = -13/2$$

$$g = -\frac{1}{2} \; ; \quad f = -2 \; ; \quad c = -13/2$$

$C \equiv (-g, -f)$ $\qquad$ $R = \sqrt{g^2 + f^2 - c}$

$\equiv \left(\frac{1}{2}, 2\right)$ $\qquad$ $= \sqrt{\frac{1}{4} + 4 + \frac{13}{2}}$

$= \sqrt{\frac{1 + 16 + 26}{4}}$ $\qquad = \frac{\sqrt{43}}{2}$

06. Find centre and radius of the circle : $3x^2 + 3y^2 - 6x + 4y - 3 = 0$

SOLUTION

$$3x^2 + 3y^2 - 6x + 4y - 3 = 0$$

$$x^2 + y^2 - 2x + \frac{4y}{3} - 1 = 0$$

On comparing with

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$2g = -2 \; ; \quad 2f = \frac{4}{3} \; ; \quad c = -1$$

$$g = -1 \; ; \quad f = -\frac{2}{3} \; ; \quad c = -1$$

$C \equiv (-g, -f)$ $\qquad$ $R = \sqrt{g^2 + f^2 - c}$

$\equiv \left(1, -\frac{2}{3}\right)$ $\qquad$ $= \sqrt{1 + \frac{4}{9} + 1}$

$= \sqrt{\frac{9 + 4 + 9}{4}}$

$= \frac{\sqrt{22}}{3}$

07. find the center and the radius of the circle : $(x - 3)(x - 5) + (y - 1)(y - 7) = 0$

$(x - 3)(x - 5) + (y - 1)(y - 7) = 0$

$x^2 - 8x + 15 + y^2 - 8y + 7 = 0$

$x^2 + y^2 - 8x - 8y + 22 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -8 \; ; \; 2f = -8 \; ; \; c = 22$

$g = -4 \; ; \; f = -4 \; ; \; c = 22$

$C \equiv (-g , -f)$ $\quad R = \sqrt{g^2 + f^2 - c}$

$\equiv (4,4)$ $\quad = \sqrt{16 + 16 - 22}$

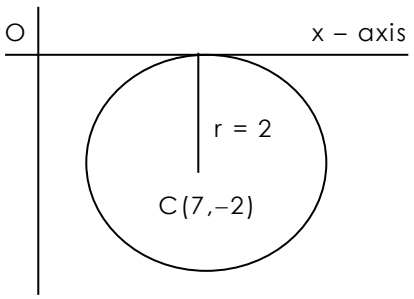
$= \sqrt{10}$

Q3.

01. find equation of the circle having centre (7,–2) and touching the x – axis

SOLUTION

$r = 2$ ..... (REFER DIAGRAM)



$C(7,-2) \; , \; r = 2$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 7)^2 + (y + 2)^2 = (2)^2$

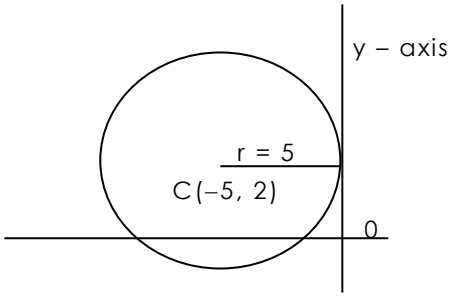
$x^2 - 14x + 49 + y^2 + 4y + 4 = 4$

$x^2 + y^2 - 14x + 4y + 49 = 0$ ..... equation of the circle

02. find equation of the circle having centre (–5,2) and touching the y – axis

SOLUTION

$r = 5$ ..... (REFER DIAGRAM)



$C(-5, 2) \; , \; r = 5$

$(x - h)^2 + (y - k)^2 = r^2$

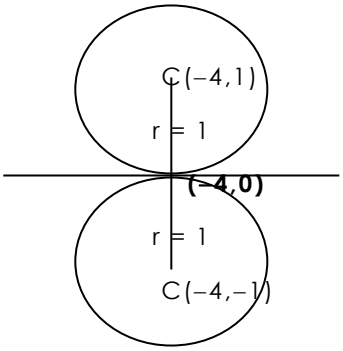
$(x + 5)^2 + (y - 2)^2 = (5)^2$

$x^2 + 10x + 25 + y^2 - 4y + 4 = 25$

$x^2 + y^2 + 10x - 4y + 4 = 0$ ..... equation of the circle

03. find equation of the circle having radius = 1 and touching the x – axis at (–4,0)

SOLUTION



CIRCLE 1 $C(-4,1)$, $r = 1$

$(x - h)^2 + (y - k)^2 = r^2$

$(x + 4)^2 + (y - 1)^2 = 1$

$x^2 + 8x + 16 + y^2 - 2y + 1 = 1$

$x^2 + y^2 + 8x - 2y + 16 = 0$

CIRCLE 2 $C(-4,-1)$, $r = 1$

$(x - h)^2 + (y - k)^2 = r^2$

$(x + 4)^2 + (y + 1)^2 = 1$

$x^2 + 8x + 16 + y^2 + 2y + 1 = 1$

$x^2 + y^2 + 8x + 2y + 16 = 0$ ans : $x^2 + y^2 + 8x \pm 2y + 16 = 0$

Q4.

01. Find circle touching both the axes and having radius 7

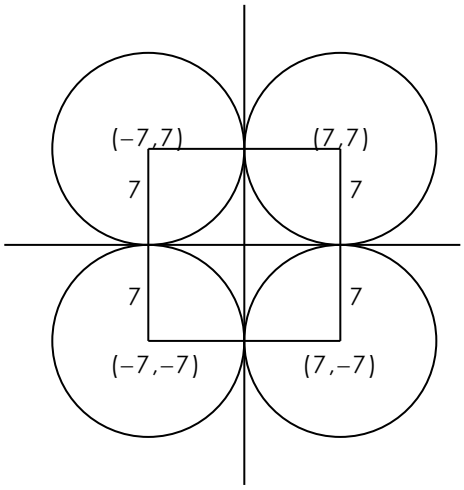
SOLUTION

Using : $(x - h)^2 + (y - k)^2 = r^2$

CIRCLE 1 $C(7,7)$, $r = 7$

$(x - 7)^2 + (y - 7)^2 = 7^2$

$x^2 + y^2 - 14x - 14y + 49 = 0$



CIRCLE 2 $C(-7,7)$, $r = 7$

$(x + 7)^2 + (y - 7)^2 = 7^2$

$x^2 + y^2 + 14x - 14y + 49 = 0$

CIRCLE 3 $C(-7,-7)$, $r = 7$

$(x + 7)^2 + (y + 7)^2 = 7^2$

$x^2 + y^2 + 14x + 14y + 49 = 0$

CIRCLE 4 $C(7,-7)$, $r = 7$

$(x - 7)^2 + (y + 7)^2 = 7^2$

$x^2 + y^2 - 14x + 14y + 49 = 0$ ans : $x^2 + y^2 \pm 14x \pm 14y + 49 = 0$

02. find equation of the circle touching both axes and passing through (1,2)

SOLUTION

STEP 1 : $CP = r$

$CP^2 = r^2$

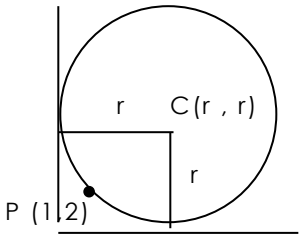
$(r-1)^2 + (r-2)^2 = r^2$

$r^2 - 2r + 1 + r^2 - 4r + 4 = r^2$

$r^2 - 6r + 5 = 0$

$(r-1)(r-5) = 0$

$r = 1 \;;\; r = 5$



STEP 2 :

$r = 1 \;;\; C(1,1)$	$r = 5 \;;\; C(5,5)$
$(x-h)^2 + (y-k)^2 = r^2$	$(x-h)^2 + (y-k)^2 = r^2$
$(x-1)^2 + (y-1)^2 = 1$	$(x-5)^2 + (y-5)^2 = 25$
$x^2 - 2x + 1 + y^2 - 2y + 1 = 1$	$x^2 - 10x + 25 + y^2 - 10y + 25 = 25$
$x^2 + y^2 - 2x - 2y + 1 = 0$	$x^2 + y^2 - 10x - 10y + 25 = 0$

03. find equation of the circle touching both axes and passing through (–9,8)

SOLUTION

STEP 1 : $CP = r$

$CP^2 = r^2$

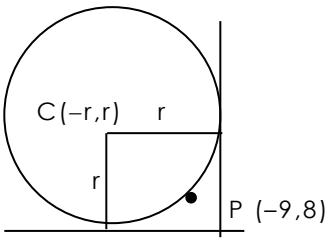
$(-r+9)^2 + (r-8)^2 = r^2$

$r^2 - 18r + 81 + r^2 - 16r + 64 = r^2$

$r^2 - 34r + 145 = 0$

$(r-5)(r-29) = 0$

$r = 5 \;;\; r = 29$



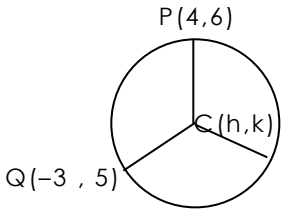
STEP 2 :

$r = 5 \;;\; C(-5,5)$	$r = 29 \;;\; C(-29,29)$
$(x-h)^2 + (y-k)^2 = r^2$	$(x-h)^2 + (y-k)^2 = r^2$
$(x+5)^2 + (y-5)^2 = 25$	$(x+29)^2 + (y-29)^2 = 841$
$x^2 + 10x + 25 + y^2 - 10y + 25 = 25$	$x^2 + 58x + 841 + y^2 - 58y + 841 = 841$
$x^2 + y^2 + 10x - 10y + 25 = 0$	$x^2 + y^2 + 58x - 58y + 841 = 0$

Q5.

01. Find equation of circle passing through (4 , 6) ; (–3 , 5) & (5 , –1)

SOLUTION



STEP 1 :

$CP = CQ$

$CP^2 = CQ^2$

$(h - 4)^2 + (k - 6)^2 = (h + 3)^2 + (k - 5)^2$

$h^2 - 8h + 16 + k^2 - 12k + 36 = h^2 + 6h + 9 + k^2 - 10k + 25$

$-8h - 12k + 52 = 6h - 10k + 34$

$18 = 14h + 2k$

$9 = 7h + k$ …… (1)

STEP 2 :

$CP = CR$

$CP^2 = CR^2$

$(h - 4)^2 + (k - 6)^2 = (h - 5)^2 + (k + 1)^2$

$h^2 - 8h + 16 + k^2 - 12k + 36 = h^2 - 10h + 25 + k^2 + 2k + 1$

$-8h - 12k + 52 = -10h + 2k + 26$

$2h - 14k = -26$

$h - 7k = -13$ …… (2)

STEP 3 : SOLVING (1) & (2)

$7 \times \quad 7h + k = 9$

$h - 7k = -13$

$49h + 7k = 63$

$h - 7k = -13$

$50h = 50$

$h = 1$

$k = 2$ $\qquad C \equiv (1 , 2)$

STEP 4 : C (1 , 2) , P(4,6)

$r = CP$

$= \sqrt{(1 - 4)^2 + (2 - 6)^2}$

$= \sqrt{25}$

$= 5$

STEP 5 : C(1 , 2) , r = 5

$(x - h)^2 + (y - k)^2 = r^2$

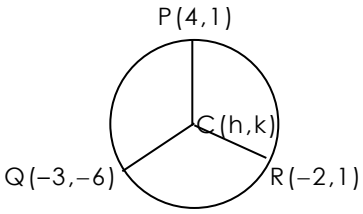
$(x - 1)^2 + (y - 2)^2 = 5^2$

$x^2 - 2x + 1 + y^2 - 4y + 4 = 25$

$x^2 + y^2 - 2x - 4y - 20 = 0$ ……. Equation of circle

02. Find equation of circle passing through (4 , 1) ; (–3 , –6) & (–2 , 1)

SOLUTION



STEP 1 :

$CP = CQ$

$CP^2 = CQ^2$

$(h - 4)^2 + (k - 1)^2 = (h + 3)^2 + (k + 6)^2$

$h^2 - 8h + 16 + k^2 - 2k + 1 = h^2 + 6h + 9 + k^2 + 12k + 36$

$-8h - 2k + 17 = 6h + 12k + 45$

$-28 = 14h + 14k$

$-2 = h + k$ …… (1)

STEP 2 :

$CP = CR$

$CP^2 = CR^2$

$(h - 4)^2 + (k - 1)^2 = (h + 2)^2 + (k - 1)^2$

$h^2 - 8h + 16 + k^2 - 2k + 1 = h^2 + 4h + 4 + k^2 - 2k + 1$

$-8h - 2k + 17 = 4h - 2k + 5$

$12 = 12h$

$h = 1$ …… (2)

STEP 3 : Solving (1) & (2)

sub $h = 1$ in (1)

$k = -3$ $\quad C \equiv (1 , -3)$

STEP 4 : $C\ (1 , -3)$, $P(4,1)$

$r = CP$

$= \sqrt{(1 - 4)^2 + (-3 - 1)^2}$

$= \sqrt{9 + 16}$

$= 5$

STEP 5 : $C(1 , -3)$, $r = 5$

$(x - h)^2 + (y - k)^2 = r^2$

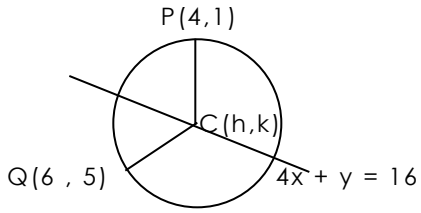
$(x - 1)^2 + (y + 3)^2 = 25$

$x^2 - 2x + 1 + y^2 + 6y + 9 = 25$

$x^2 + y^2 - 2x + 6y - 15 = 0$ ……. Equation of circle

03. Find equation of circle passing through (4 , 1) ; (6 , 5) & whose center lies on $4x + y = 16$

SOLUTION



STEP 1 :

Since C(h , k) lies on $4x + y = 16$

$4h + k = 16$ …… (1)

STEP 2 :

$CP = CQ$

$CP^2 = CQ^2$

$(h - 4)^2 + (k - 1)^2 = (h - 6)^2 + (k - 5)^2$

$h^2 - 8h + 16 + k^2 - 2k + 1 = h^2 - 12h + 36 + k^2 - 10k + 25$

$-8h - 2k + 17 = -12h - 10k + 61$

$4h + 8k = 44$

$h + 2k = 11$ …… (2)

STEP 3 : $2 \times (4h + k = 16)$ $\qquad$ $h + 2k = 11$

$8h + 2k = 32$

$h + 2k = 11$

$7h = 21$

$h = 3$

$k = 4$ $\qquad$ $C \equiv (3 , 4)$

STEP 4 : C (3 , 4) , P(4,1)

$r = CP$

$= \sqrt{(3 - 4)^2 + (4 - 1)^2}$

$= \sqrt{1 + 9}$

$= \sqrt{10}$

STEP 5 : C(3 , 4) , $r = \sqrt{10}$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 3)^2 + (y - 4)^2 = 10$

$x^2 - 6x + 9 + y^2 - 8y + 16 = 10$

$x^2 + y^2 - 6x - 8y + 15 = 0$ …… Equation of circle

Q6. Show that following pair of circles touch each other . Find the point of contact

03. $x^2 + y^2 - 4x + 10y + 20 = 0$; $x^2 + y^2 + 8x - 6y - 24 = 0$

STEP 1 :

$x^2 + y^2 - 4x + 10y + 20 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -4$; $2f = 10$

$g = -2$; $f = 5$; $c = 20$

$C_1 \equiv (-g , -f)$ $\quad r_1 = \sqrt{g^2 + f^2 - c}$

$\equiv (2 , -5)$ $\quad = \sqrt{4 + 25 - 20}$

$= 3$

STEP 2 :

$x^2 + y^2 + 8x - 6y - 24 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = 8$; $2f = -6$

$g = 4$; $f = -3$; $c = -24$

$C_2 \equiv (-g , -f)$ $\quad r_2 = \sqrt{g^2 + f^2 - c}$

$\equiv (-4 , 3)$ $\quad = \sqrt{16 + 9 + 24}$

$= 7$

STEP 3 : $C_1 \equiv (2 , -5)$; $C_2 \equiv (-4 , 3)$

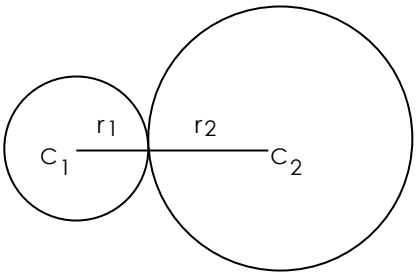
$C_1C_2 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(2 + 4)^2 + (-5 - 3)^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$= 10$



STEP 4 : $C_1 C_2 = r_1 + r_2$. Hence the circles touch each other **externally** .

STEP 5 : P divides C_1C_2 internally in the ratio 3 : 7

C_1 $\quad 3 \quad$ P $\quad 7 \quad$ C_2

$(2 , -5)$ $\quad (x,y) \quad$ $(-4 , 3)$

Using section formula (internal division)

$x = \dfrac{mx_2 + nx_1}{m + n}$ $\qquad y = \dfrac{my_2 + ny_1}{m + n}$

$= \dfrac{3(-4) + 7(2)}{3 + 7}$ $\qquad = \dfrac{3(3) + 7(-5)}{3 + 7}$

$= \dfrac{2}{10}$ $\qquad = \dfrac{-26}{10}$ $\qquad P \equiv \left(\dfrac{1}{5} , \dfrac{-13}{5} \right)$

04. $x^2 + y^2 + 4x - 12y + 4 = 0$; $x^2 + y^2 - 2x - 4y + 4 = 0$

STEP 1 :

$x^2 + y^2 + 4x - 12y + 4 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = 4$; $2f = -12$

$g = 2$; $f = -6$; $c = 4$

$C_1 \equiv (-g , -f) \equiv (-2 , 6)$

$r_1 = \sqrt{g^2 + f^2 - c} = \sqrt{4 + 36 - 4} = 6$

STEP 2 :

$x^2 + y^2 - 2x - 4y + 4 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -2$; $2f = -4$

$g = -1$; $f = -2$; $c = 4$

$C_2 \equiv (-g , -f) \equiv (1 , 2)$

$r_2 = \sqrt{g^2 + f^2 - c} = \sqrt{1 + 4 + 4} = 1$

STEP 3 : $C_1 \equiv (-2 , 6)$; $C_2 \equiv (1 , 2)$

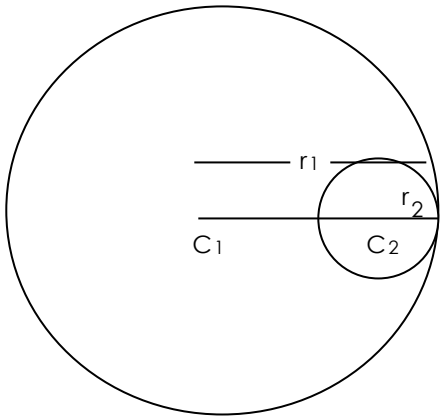
$C_1C_2 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(-2 - 1)^2 + (6 - 2)^2}$

$= \sqrt{9 + 16}$

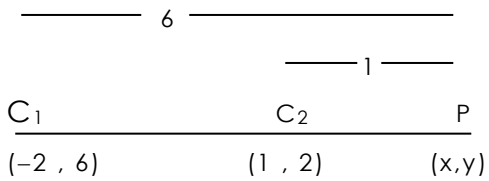
$= \sqrt{25}$

$= 5$



STEP 4 : $C_1C_2 = r_1 - r_2$. Hence the circles touch each other **internally**

STEP 5 : P divides C_1C_2 externally in the ratio 6 : 1



Using section formula (internal division)

$x = \frac{mx_2 - nx_1}{m - n} = \frac{6(1) - 1(-2)}{6 - 1} = \frac{8}{5}$

$y = \frac{my_2 - ny_1}{m - n} = \frac{6(2) - 1(6)}{6 - 1} = \frac{6}{5}$

$P \equiv \left(\frac{8}{5} , \frac{6}{5}\right)$